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STUDY OF STRUCTURE OF THE MASS GAP BETWEEN TWO SPIN MULTIPLETS

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Abstract

Studying our semirelativistic potential model and the numerical results, which succeeds in predicting and reproducing recently discovered higher resonances of D, D_s, B, and B_s, we find a simple expression for the mass gap between two spin multiplets of heavy-light mesons, $(0^-, 1^-)$ and $(0^+, 1^+)$. The mass gap between chiral partners defined by $\Delta M = M(0^+) - M(0^-)$ and/or $M(1^+) - M(1^-)$ is given by $\Delta M = M(0^+) - M(0^-) = M(1^+) - M(1^-) \approx \Lambda_Q - m_q$ in the limit of heavy quark symmetry. We also study the case including $1/m_Q$ corrections.

1 Introduction

The discovery of the narrow D_{sJ} particles by BaBar [1] and CLEO [2] and soon confirmed by Belle [3] immediately reminded people an effective theory

approach proposed by Nowak et al. and others [4, 5, 6, 7]. From this effective theory, they derived the Goldberger-Treiman relation for the mass gap between chiral partners $0^+(1^+)$ and $0^-(1^-)$ instead of the heavy meson mass itself and predicted the mass gap to be around $\Delta M = g_\pi f_\pi \approx 349$ MeV, where g_π is the coupling constant for $0^+ \to 0^- + \pi$ and f_π is the pion decay constant.

Since this mass gap between chiral partners in the case of D_s agrees well with the experiments (around 350 MeV), people thought that underling physics may be explained by their $SU(3)$ effective Lagrangian [8, 9]. However, when $(0^+, 1^+)$ for D meson were found by Belle and FOCUS, and later reanalyzed by CLEO, their explanation needs to be modified. Furthermore, what they originally predicted could not be identified as any of heavy meson multiplets for D, D_s, B, and B_s. In other words, the forumula can be applied equally for any of these heavy meson multiplets. Thus, it is required to find the mass gap formula, if it exists, which agrees well with the experiments and explains the physical ground of its formula.

In this paper, using our semirelativistic potential model, we first give our formula for the mass gap between chiral partners $0^+(1^+)$ and $0^-(1^-)$ for *any* heavy meson, D, D_s, B, and B_s, among which the known mass gaps, i.e., the ones for D and D_s, agree well with the experiments although there is some ambiguities for D meson data. Next we show how this mass gap depends on a light quark mass m_q for $q = u, d$, and s, where we neglect the difference between u and d quarks. Our formula naturally explain that the mass gap for D is larger than that for D_s and predict the mass gaps for B and B_s.

2   Semirelativistic Quark Potential Model and Structure of Mass Gap

Mass for the heavy meson X with the spin and parity, j^P, is expressed in our formulation as [12]

$$M_X(j^P) = m_Q + E_0^k(m_q) + O\left(1/m_Q\right),\tag{1}$$

where the quantum number k is related to the total angular momentum j and the parity P for a heavy meson as

$$j = |k| - 1 \;\; \text{or} \;\; |k|, \quad P = \frac{k}{|k|}(-1)^{|k|+1}, \quad E_0^k(m_q) = E_0(j^P, m_q).\tag{2}$$

To begin with, we study the heavy meson mass without $1/m_Q$ corrections so that we can see the essence of the mass gap. States with the same $|k|$ value are degenerate in a pure chiral limit and without confining scalar potential, which is defined as $m_q \to 0$ and $S(r) \to 0$ [13]. We consider the scenario that a chiral symmetry breaking and a confinement take place in two steps. First the degeneracy is broken due to gluon fields when $S(r)$ is turned on and confines quarks into heavy mesons but keeping vanishing light quark mass intact. In fact, in this limit our model gives the mass gap between two spin multiplets $\Delta M \approx 300$ MeV as follows;

$$
\begin{aligned}
\Delta M &= E_0(1^+, 0) - E_0(1^-, 0) = E_0(0^+, 0) - E_0(0^-, 0) \\
&= 295.1 \text{ MeV} \quad \text{for } D, \text{ and } D_s, \\
&= 309.2 \text{ MeV} \quad \text{for } B, \text{ and } B_s,
\end{aligned} \tag{3}
$$

This gap is mainly due to gluon fields which confines quarks into heavy mesons. It is interesting that obtained values are close to $\Lambda_{\mathrm{QCD}} \approx 300$ MeV. Next, turning on a light quark mass which explicitly breaks a chiral symmetry, we have $SU(3)$ flavor breaking pattern of the mass levels, i.e., mass of D becomes different from that of D_s with the same value of j^P. Since we assume $m_u = m_d$, there still remains $SU(2)$ iso-spin symmetry. Note that even after chiral symmetry is broken, there is still degeneracy between members of a spin multiplet due to the heavy quark symmetry, i.e., $SU(2)_f \times SU(2)_{\mathrm{spin}}$ symmetry, with $SU(2)_f$ rotational flavor symmetry and $SU(2)_{\mathrm{spin}}$ rotational spin symmetry. By using the optimal values of parameters in Ref. [14], which is listed in Table 1, degenerate masses without $1/m_Q$ corrections for D, D_s and B, B_s mesons are calculated and presented in Table 2. Furthermore, by changing m_q from 0 to 0.2 GeV, we have calculated the m_q dependence of ΔM_0 and have obtained Fig. 1, in which ΔM_0 is linearly decreasing with m_q. From Fig. 1, we find that the mass gap between two spin multiplets for a heavy meson X can be written as

$$
\begin{aligned}
\Delta M_0 &= M_X(0^+) - M_X(0^-) = M_X(1^+) - M_X(1^-) = g_0 \Lambda_{\mathrm{Q}} - g_1 m_q, \tag{4} \\
\Lambda_Q &= 300 \text{ MeV}, \begin{cases} g_0 = 0.9836, \ g_1 = 1.080, & \text{for } D/D_s \\ g_0 = 1.017, \ g_1 = 1.089, & \text{for } B/B_s \end{cases}, \tag{5}
\end{aligned}
$$

where the values of g_0, and g_1 are estimated by fitting the optimal line with Fig. 1. Since both g_0 and g_1 are very close to 1, we conclude that the mass
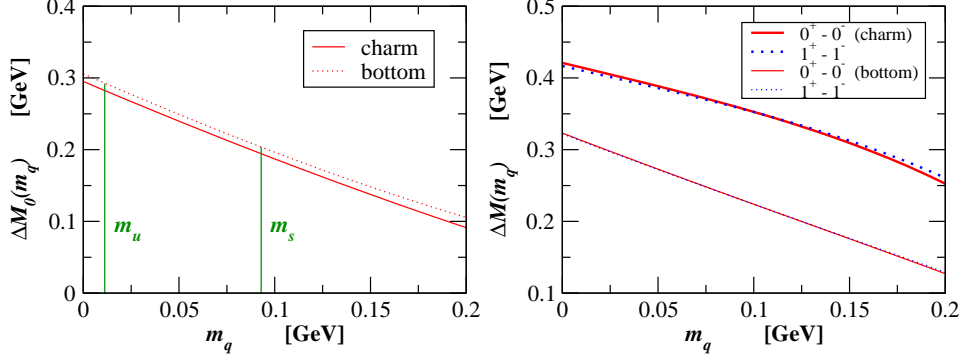
Figure 1: Plots of the mass gap between two spin multiplets. Light quark mass dependence is given. The horizontal axis is light quark mass m_q and the vertical axis is the mass gap. Both the heavy quark limit ΔM_0 (Left) and the one with $1/m_Q$ corrections ΔM (Right) are given.

gap is essentially given by

$$\Delta M_0 = \Lambda_Q - m_q \tag{6}$$

Though the physical ground of this result is out of scope at present, Eq. (6) is serious, since it is very different from the one of an effective theory approach which gives the relation,

$$\Delta M_0 = g_\pi \left(\langle \sigma \rangle + m_q \right). \tag{7}$$

where g_π is the Yukawa coupling constant between the heavy meson and a chiral multiplet and is taken to be $g_\pi = 3.73$ in [8], and $\langle \sigma \rangle = f_\pi$. This expression is obtained in the heavy quark symmetric limit and should be compared with our Eq. (6). Instead of minus sign for the term m_q that we obtained, the authors of [5] obtained plus sign as shown in the above equation. The same result is obtained even if we use the nonlinear Σ model [8]. The result given by Eq. (6) is exact when $\mathcal{O}\left(1/m_Q\right)$ terms are neglected. As we will see later, since $1/m_Q$ corrections are nearly equal to each other for two spin doublets, the above equation (6) between two spin multiplets holds approximately even with $1/m_Q$ corrections.

The reason why the mass gap can be written like Eq. (5) or (6) is explained in our formulation. (See the details in Refs. [11] and [12].)

Table 1: Optimal values of parameters.

Params.	α_s^c	α_s^b	a (GeV^{-1})	b (GeV)
	0.261±0.001	0.393±0.003	1.939±0.002	0.0749±0.0020
	$m_{u,d}$ (GeV)	m_s (GeV)	m_c (GeV)	m_b (GeV)
	0.0112±0.0019	0.0929±0.0021	1.032±0.005	4.639±0.005
	# of data	# of parameter	total χ^2/d.o.f	
	18	8	107.55	

Table 2: Degenerate masses of model calculations and their mass gap between $0^+(1^+)$ and $0^-(1^-)$ for $n = 1$.

	$M_0(D)$	$M_0(D_s)$	$M_0(B)$	$M_0(B_s)$
$0^-/1^-$	1784	1900	5277	5394
$0^+/1^+$	2067	2095	5570	5598
$0^+(1^+) - 0^-(1^-)$	283	195	293	204

3 $1/m_Q$ Corrections

Next let us study the case when $1/m_Q$ corrections to the mass gap are taken into account. Part of the results is given in [15]. In Table 3, we give our numerical results in the cases of $n = 1$ and $n = 2$ (radial excitations). Values in brackets are taken from the experiments. Our values seem to agree with the experimental ones though the fit is not as good as the case for the absolute values of heavy meson masses. We assume the form of the mass gap with the $1/m_Q$ corrections as follows.

$$\Delta M = \Delta M_0 + \frac{c + d \cdot m_q}{m_Q}. \tag{8}$$

Using Eq.(4) for D and D_s mesons, i.e. $\Delta M_0 = g_0 \Lambda_Q - g_1 m_q = 295.1 - 1.080 m_q$, we obtain the values of the parameters c and d for D/D_s mesons given in Table 3, which are given by

$$c = 1.28 \times 10^5 \text{ MeV}^2, \quad d = 4.26 \times 10^2 \text{ MeV}. \tag{9}$$

The term c/m_Q lifts the constant $g_0 \Lambda_Q$ about 100 MeV and the term d/m_Q gives deviation from -1 to the coefficient for m_q in the case of D/D_s.

Applying this formula, Eq. (8), to the case for B/B_s with $m_Q = m_b$, we

Table 3: Model calculations of the mass gap. Values in brackets are taken from the experiments. Units are MeV.

Mass gap ($n = 1$)	$\Delta M(D)$	$\Delta M(D_s)$	$\Delta M(B)$	$\Delta M(B_s)$
$0^+ - 0^-$	414 (441)	358 (348)	322	239
$1^+ - 1^-$	410 (419)	357 (348)	320	242

($n = 2$)	$\Delta M(D)$	$\Delta M(D_s)$	$\Delta M(B)$	$\Delta M(B_s)$
$0^+ - 0^-$	308	274	206	160
$1^+ - 1^-$	350	327	216	171

obtain the mass gap as follows.

$$
\begin{aligned}
B(0^+) - B(0^-) &\approx B(1^+) - B(1^-) \approx 322, \\
B_s(0^+) - B_s(0^-) &\approx B_s(1^+) - B_s(1^-) \approx 240 \quad \text{MeV}, \quad (10)
\end{aligned}
$$

which should be compared with our model calculations, 321 and 241 MeV, in Table 3. Thus the linear dependence of the mass gap on m_q is also supported in the case where the $1/m_Q$ corrections are taken into account. The calculated m_q dependence of ΔM with $1/m_Q$ corrections is presented in Fig. 2, for $0 < m_q < 0.2 \text{GeV}$.

4 Miscellaneous Phenomena

Global Flavor $SU(3)$ Recovery – Looking at the mass levels of 0^+ and 1^+ states for the D and D_s mesons, one finds that mass differences between D and D_s becomes smaller compared with those of the 0^- and 1^- states. This can be seen from Table 4 and was first discussed in Ref. [16] by Dmitrašinović. He claimed that considering D_{sJ} as a four-quark state, one can regard this phenomena as flavor $SU(3)$ recovery. However, in our interpretation, this is not so as we have seen that this is caused by the mass gap dependency on a light quark mass, m_q, as shown in Fig. 1. That is, when the mass of D meson is elevated largely from the $0^-/1^-$ state to the $0^+/1^+$ state, the mass of D_s meson is elevated by about 100 MeV smaller than that of $0^-/1^-$ as one can see from Fig. 1. In our interpretation, the $SU(3)$ is not recovered since the light quark masses of $m_u = m_d$ and m_s do not change their magnitudes when the transition from $0^-/1^-$ to $0^+/1^+$ occurs, and their values remain to be $m_{u(d)} = 11.2$ MeV and

Table 4: D/D_s meson mass spectra for both the calculated and experimentally observed ones. Units are MeV.

$^{2s+1}L_J(J^P)$	$M_{\text{calc}}(D)$	$M_{\text{obs}}(D)$	$M_{\text{calc}}(D_s)$	$M_{\text{obs}}(D_s)$
$^1S_0(0^-)$	1869	1867	1967	1969
$^3S_1(1^-)$	2011	2008	2110	2112
$^3P_0(0^+)$	2283	2308	2325	2317
$"^3P_1"(1^+)$	2421	2427	2467	2460

Table 5: B/B_s meson mass spectra for both the calculated and experimentally observed ones. Units are MeV.

$^{2s+1}L_J(J^P)$	$M_{\text{calc}}(B)$	$M_{\text{obs}}(B)$	$M_{\text{calc}}(B_s)$	$M_{\text{obs}}(B_s)$
$^1S_0(0^-)$	5270	5279	5378	5369
$^3S_1(1^-)$	5329	5325	5440	−
$^3P_0(0^+)$	5592	−	5617	−
$"^3P_1"(1^+)$	5649	−	5682	−

$m_s = 92.9$ MeV, respectively, as presented in Table 1.

Mass Gap of Heavy Baryons – When we apply our formula to the heavy-light baryons which include two heavy quarks, (ccs), (ccu), (bcs), (bcu), (bbs), and (bbu), mass gaps between two pairs of baryons, like (ccs) and (ccu), will be given by Eq. (6) in the heavy quark symmetric limit and by Eq. (8) with $1/m_Q$ corrections where we have to replace m_Q with $m_{Q_1} + m_{Q_2}$. Here the isospin symmetry is respected since in our model $m_u = m_d$. This speculation is legitimized since QQ pair can be considered to be 3^* expression in the color $SU(3)$ space so that the baryon like QQq can be regarded as a heavy-light meson and our arguments expanded in this paper can be applied [17, 18].

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